A perturbed star emits Gravitational Waves at the frequencies of the quasi-normal modes (Complex eigenfrequencies)*:*

The quasi-normal modes are solutions of the axial and polar equations that satisfy the following boundary conditions:

Pure outgoing wave at radial infinity

All perturbed functions have a regular behavior at r=0

The interior and exterior solutions match continuously at the surface of the star

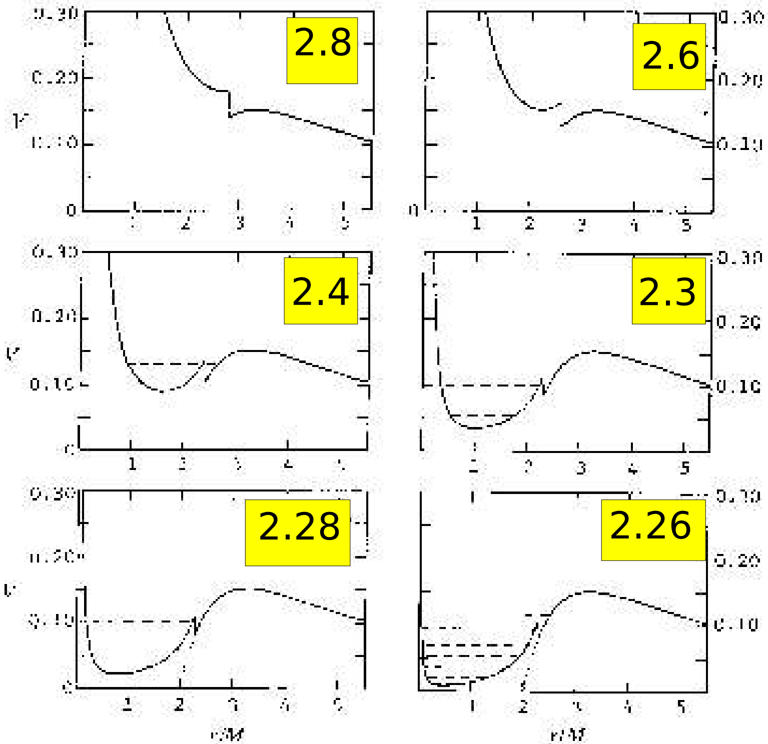
In GR there exist family of modes that do not have a newtonian counterpart

Quasi-normal modes for the Axial Perturbations

$$\begin{cases} \frac{d^2 Z^{ax}}{dr_*^2} + [\omega^2 - V_\ell^{ax}(r)]Z^{ax} = 0, \\ V^{ax}(r) = \frac{e^{2\nu}}{r^3}[\ell(\ell+1)r + r^3(\epsilon - p) - 6m(r)]. \end{cases}$$

where $r_* = \int_0^r e^{-\nu+\mu_2} dr$,

Let us plot the potential barrier for a homogeneous stars. for different values of the compactness $\frac{R}{M}$



S.Chandrasekhar, V.Ferrari, Proc. R. Soc. Lond. A432, 1990

These are pure modes of the gravitational field

Even when the star is less compact there are modes of the grav field: the w-modes
Typical frequencies 8-12 kHz
damping times 0.01-0.1 ms

They exist also for polar perturbations

K.D.Kokkotas, B.F. Schutz M.N.R.A.S. 255 (1992)
M.Leins, H.P. Noellert, M.H.Soffel
Phys. Rev.D48 (1993)

These modes do not exist in newtonian theory

The polar quasi-normal modes belong to different classes:

G-modes: main restoring force is the buoyancy force

F-mode: has an inter-mediate character of p- and g-mode

P-modes: main restoring force is the pressure

W-modes: pure space-time modes (only in GR)

Inertial modes (r-modes) :main restoring force is the Coriolis force (if the star rotates)

$$\ldots\omega_{gn} < \ldots < \omega_{g0} < \omega_{f} < \omega_{p1} < \ldots < \omega_{pn} \ldots < \omega_{w1} < \ldots \omega_{wn}$$

g- modes appear if there are thermal or composition gradients

When a fluid element is displaced by its equilibrium position, it has a radial acceleration given by

$$a = -\frac{e^{-\lambda/2}}{(\epsilon + p)\gamma p}\left(-\frac{dp}{dr}\right) S(r)\,\Delta r$$

$$S(r) = \frac{dp}{dr}\left(1 - \frac{c_s^2}{c_0^2}\right)$$

Schwarzschild discriminant

$$c_s^2 = \left(\frac{\partial p}{\partial \epsilon}\right)_{\mathrm{s,Y_L}}$$

sound velocity

$$c_0^2 = \frac{dp/dr}{d\epsilon/dr}$$

equilibrium velocity

If **S (r) > 0** the fluid element oscillate about the equilibrium position: a **g-mode** appears

If **S (r) < 0** there is a convective instability

If **S (r) = 0** (as it is for a chemically homogeneous, **T=0** star) **g-modes** degenerate to zero frequency

NON RADIAL STELLAR OSCILLATIONS

A neutron star born in a gravitational collapse wildly oscillates while contracting and cooling down

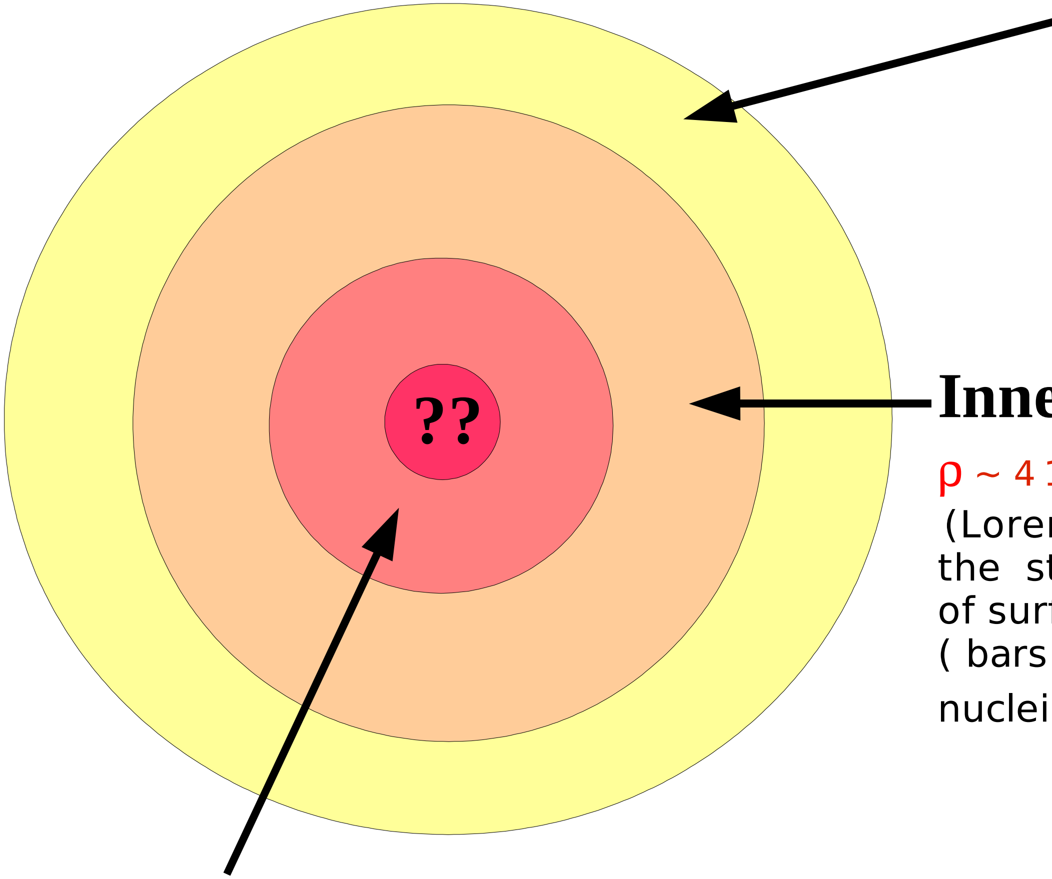
At some point of the evolution the density of the inner core may reach a critical value needed to ignite a phase transition to quark matter -> mini collapse -> stellar oscillations

Starquakes, glitches, interactions with a stellar companion etc, would perturb the star and set it in oscillations

STELLAR OSCILLATIONS OCCUR IN A VARIETY OF ASTROPHYSICAL PROCESSES

What are the characteristic frequencies?
Do they carry information on nuclear and thermodynamical processes?
How much energy should be stored into the modes to detect the emitted gravitational signal ?

Neutron Star Structure (not in scale!)



Outer crust (~ 300 m):

$\rho \sim 10^7 - 4\ 10^{11}$ g/cm^3

Baym-Pethick-Sutherland EOS:
heavy nuclei lattice immersed in an electron gas

Inner crust (~ 500 m):

$\rho \sim 4\ 10^{11} - 2.67 \times 10^{14}$ g/cm^3

(Lorenz-Ravenhall-Pethick EOS)
the structure is determined by a minimization of surface and coulomb energies
(bars of proton rich matter -> slabs -> ...)
nuclei + neutrons + e^-

Core (~ 10 km) : $\rho > \rho_0$ $\rho_0 = 2.67 \times 10^{14}$ g/cm^3

$n \dashrightarrow p + \mu + \nu_\mu$ n + p + e^- + μ in β-equilibrium

Quasi-normal modes carry information on the equation of state of matter at supranuclear densities.

Benhar, Ferrari, Gualtieri Phys. Rev.D to appear

We construct models of Neutron Stars with different EOS in the inner core

$\rho > \rho_0$ $\rho_0 = 2.67 \times 10^{14}\ g/cm^3$ equilibrium density of nuclear matter

1) Many body Theory:

APR2: uniform fluid of n p e^- μ^- , 3-body interaction phenomenological Hamiltonian 2-body potential= Argonne v18, 3-body potential=Urbana IX relativistic corrections are taken into account, ground state energy -> variational tech.
Akmal A., Pandharipande V.R, Ravenhall D.G., Phys. Rev C58, 1998

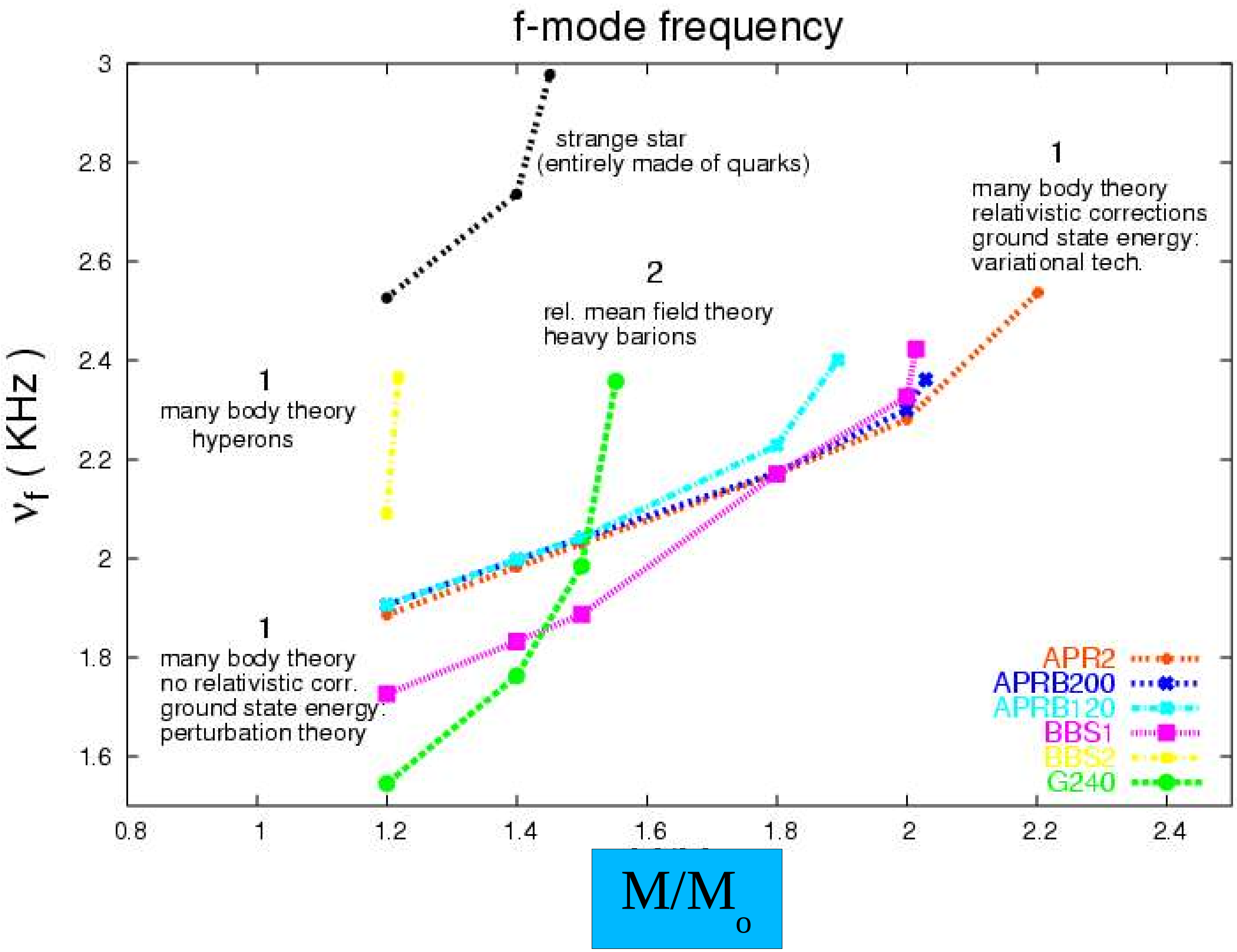
BBS1: 3-body potential= Urbana VII
(no relativistic corrections , ground state energy -> G-matrix perturbation theory
Baldo M., Burgio G.F., Schulze H,J, Phys. Rev. C61, 2000

BBS2: same as **BBS1**+ **heavy barions** Σ^{-} and Λ^0 (no relativistic corrections)
Baldo M., Burgio G.F., Schulze H,J, *Phys. Rev. C61, 2000*

APRB200, APRB120 = APR2 + interacting quarks confined to a finite region (the bag) whose volume is limited by a pressure **B** said the bag constant
(**B**=200 or 120 MeV/fm^3, α_s=0.5 m_s=150 MeV)

1) Relativistic Mean Field Theory:

G240: e^- μ^- and the complete octet of baryons ($\Sigma^{0\pm}$, $\Lambda^{0\pm}$ $\Xi^{0\pm}$ *) N.K.Glendenning*
in “Compact Stars”, Springer Verlag 2000

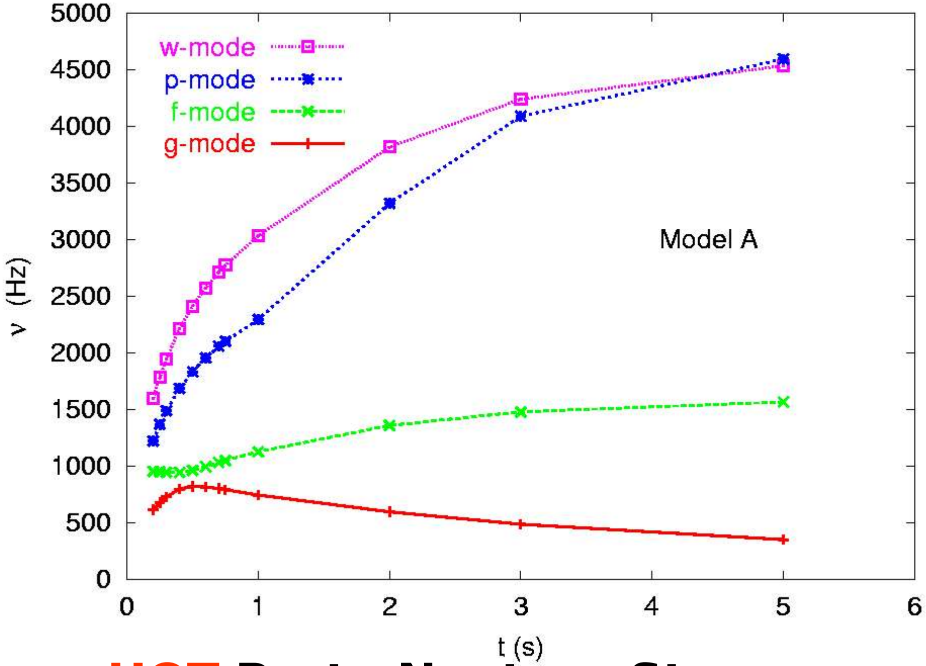


- Comparing **APR2** and **APRB200** and **APRB120** we can say nothing about the presence of a quark core

- The transition to hyperonic matter (**BBS2**) produces a significant softening of the EOS
- Similar effect in strange stars
- **G240** : uses the relativistic mean field approximation: very different dynamics

For a newly born, hot neutron Stars

V. Ferrari, G. Miniutti, J. Pons, MNRAS 2003 :



For a HOT Proto-Neutron Star:

At very early time the frequencies of the f- p1- and w1- modes **are much lower** than those of a cold Neutron Star.

In a hot, proto-neutron star high frequency g-modes appear due to entropy- and composition gradients

For a cold Neutron Star:

$\nu_f \approx$ **1.5-3 kHz** **scales as** $\propto (M/R^3)^{1/2}$

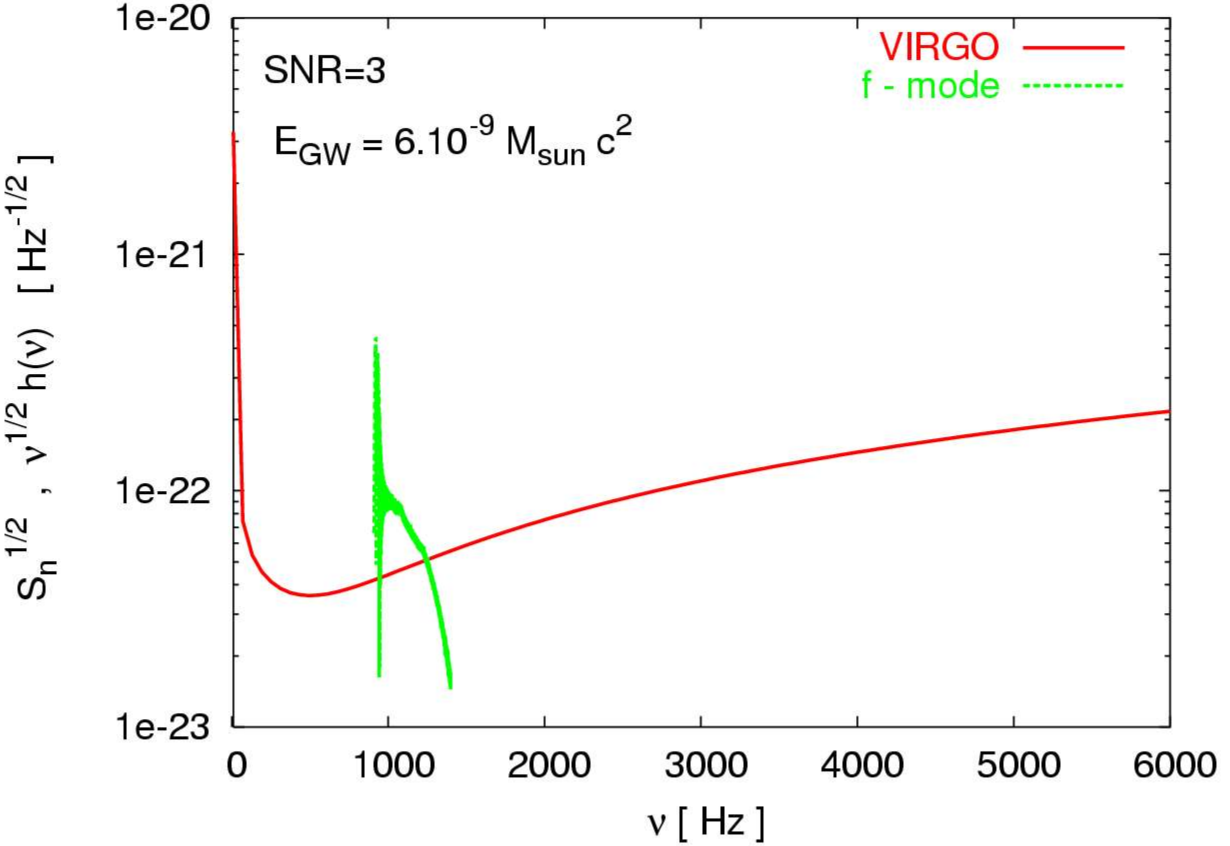
$\nu_{p1} \approx$ **5-7 kHz**

$\nu_{w1} \approx$ **8-11 kHz**

very low frequency g-modes associated to the crust

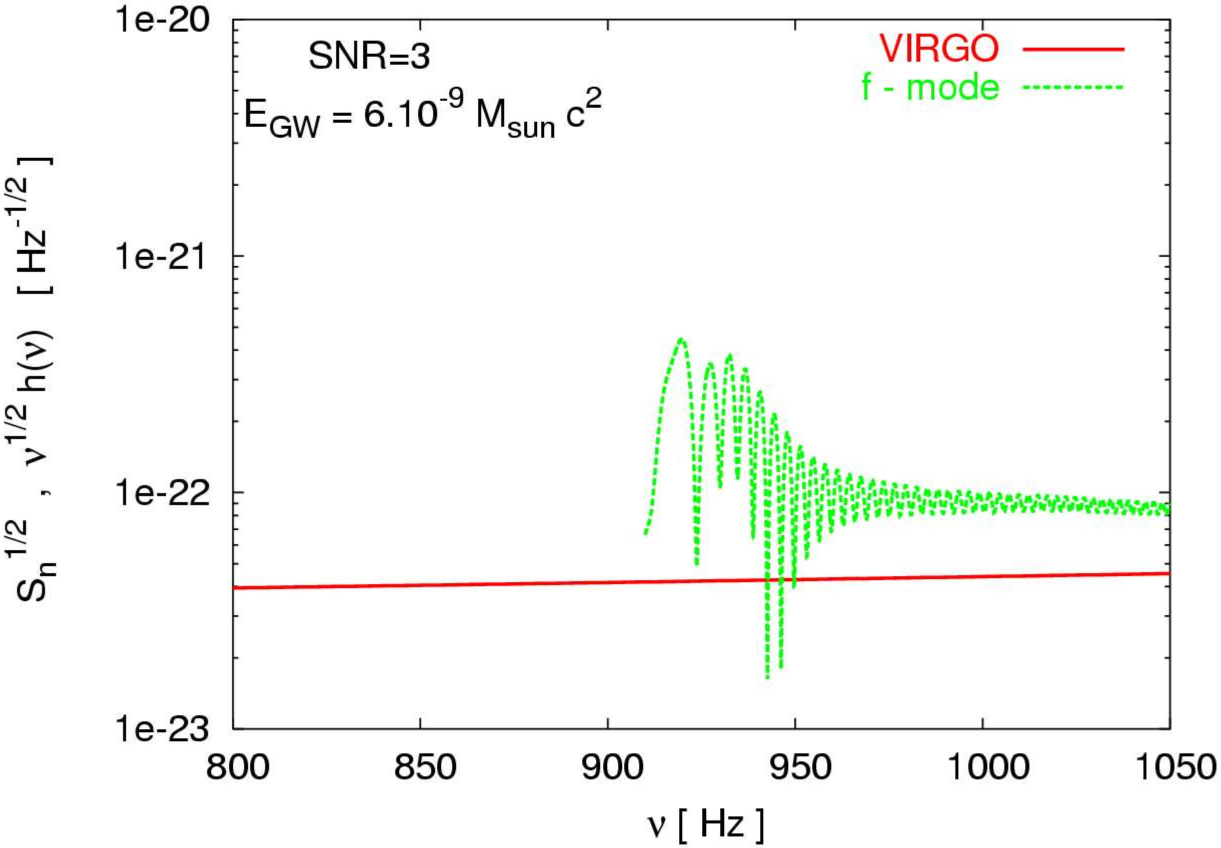
Knowing how the frequency and the damping time of the modes changes as the star evolves and cools down, we can reconstruct the emitted GW signal

VIRGO + signal emitted by a newly born NS (source in the Galaxy)



Gravitational waves from newly born, hot neutron Stars V. Ferrari, G. Miniutti, J. Pons, MNRAS 2003 :

dual



Recent simulations (Shibata&Uryu 2002) show that short-lived, supra-massive Proto-Neutron Stars could also be formed in the merger of two NSs with comparable mass and low compactness: in this case the energy radiated in GW would be much higher, up to ~ $\mathbf{10^{-2}\ M_{sun}\, c^2}$

The results of our study should also be applicable to the hot star produced by the merging, and in that case much more energy would be emitted, increasing chances of detection.

In Conclusion:

Quasi-Normal Mode frequencies carry information on the evolution of ProtoNeutron Stars and on the equation of state of matter in the interior of Neutron Stars at densities and pressures that are unaccessible by experiments on Earth.

... and if a star rotates? The axial and polar perturbations are no longer decoupled:

a lot of troubles !!!

You will learn more in the next talk